Parametric resonance of a vortex in an active medium

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Experimental observation of parametric resonance of a vortex in an active medium is reported. Unlike the parametric resonance in conservative systems, no parametric pumping of energy is involved here, making the resonance especially interesting. An alternating electric-field with frequency, equal to double frequency of the vortex rotation, has induced the vortex drift in the Belousov-Zhabotinsky chemical active medium. The drift velocity was about $\frac{1}{5}$ of the vortex drift velocity in constant electric field with the same amplitude. The direction of the drift did not coincide with the direction of the electric field and could be arbitrarily chosen by changing the phase shift between the electric-field oscillations and the vortex rotation. No effects were observed at a frequency equal to the frequency of vortex rotation, as well as at nonresonant frequencies.

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Rotating vortices (spiral waves) in active excitable media may play either a constructive role as in morphogenetic processes of social amoebae Dyctiostelium discoideium [1,2] or a destructive role as in the cardiac muscle [3] or the retina [4].

The control of vortices is a question of interest for all excitable media. The behavior of vortices was controlled by the excitation waves [5], light [6,8], and electric field [9,10]. Resonant behavior of vortices was investigated in Ref. [1], where a resonant frequency equal to the frequency of the vortex rotation was found. In this paper, we describe an example of parametric resonance of a vortex in an active medium, the resonant frequency being double the frequency of the vortex rotation.

Parametric resonance attracted attention as early as the medieval age, when a giant pendulum was parametrically pumped by periodic changes of its length during religious ceremonies in the Cathedral of Santiago de Compostela, four centuries before Galileo and Huyghens started analyzing the pendulum [11,12]. The phenomenon has been investigated in many conservative systems [13,14], the mechanism being the parametric pumping of energy. No parametric pumping of energy takes place in excitable active media.

The experimental study of this phenomenon requires high sensitivity and long-time observations without a significant change in the chemical composition of the medium, as provided by continuously stirred tank reactors (CSTR). Our experiment was performed in an electrochemical reactor [9,10,15-19] analogous to that in Ref. [20]. The main part of the open reactor was a small glass plate (20 mm long, 20 mm wide, where the gel was set) placed between two continuously stirred flow tanks with identical fixed feed concentrations, so concentrations of the reactants were kept constant throughout the experiment. The vortex, originally rotating rigidly around a circular core, was affected by electric pulses of alternating polarity, controlled by a computer. Positive electric pulses (duration 5.5 s, current density 3 mA/mm^2) were delivered when the vortex tip was oriented toward north and south, and negative pulses of the same amplitude and duration when it was oriented toward west and east. The reaction was run in a 1-mm-thick silica gel with immobilized ferriin catalyst [21] covered with a solution (1 mm) of the Belousov-Zhabotinsky (BZ) reaction [22] components: $0.17M$ $NaBrO_3$, $0.17M$ $CH_2(COOH)_2$, and $0.17M$ H_2SO_4. The temperature was controlled and kept equal to 20 °C.

At parametric resonant frequency $f=2f_@$ (where $f_@=\frac{1}{37}$ s^{-1} is the frequency of the vortex), the vortex was observed to drift. Figure 1(a) shows the vortex after 40 min of the experiment, and its displacement. The drift velocity is 0.12 mm/min, about $\frac{1}{20}$ of the velocity of free wave propagation (without electric field), and $\frac{1}{5}$ of the drift velocity induced by a constant electric field of the same intensity.

The evolution of the position of the vortex tip in the experiment is shown in Fig. 1(b) where character B marks the time when the photo of Fig. 1(a) was taken. Small oscillations of the position of the tip during its global linear displacement correspond to the rotation of the tip around the core. This provides an estimation of the core diameter, about 0.3 mm. At $t=40$ min, the phase of the electric field was increased by 90° (character B on the graph), and the drift direction has changed to the opposite one.

Numerical simulations of the two-variable reaction-diffusion (Oregonator) model [16,23–26] fit well to the experiment [Fig. 1(c)]. Its equations are

$$\partial u/\partial t=F(u,v)/\varepsilon+D_u\nabla^2 u+ME\partial u/\partial x \ ,$$
$$\partial v/\partial t=\phi(u,v)+D_v\nabla^2 v \ , \qquad (1)$$

where the term $ME\ \partial u/\partial x$ describes the effect of the electric field parallel to the X axis, functions

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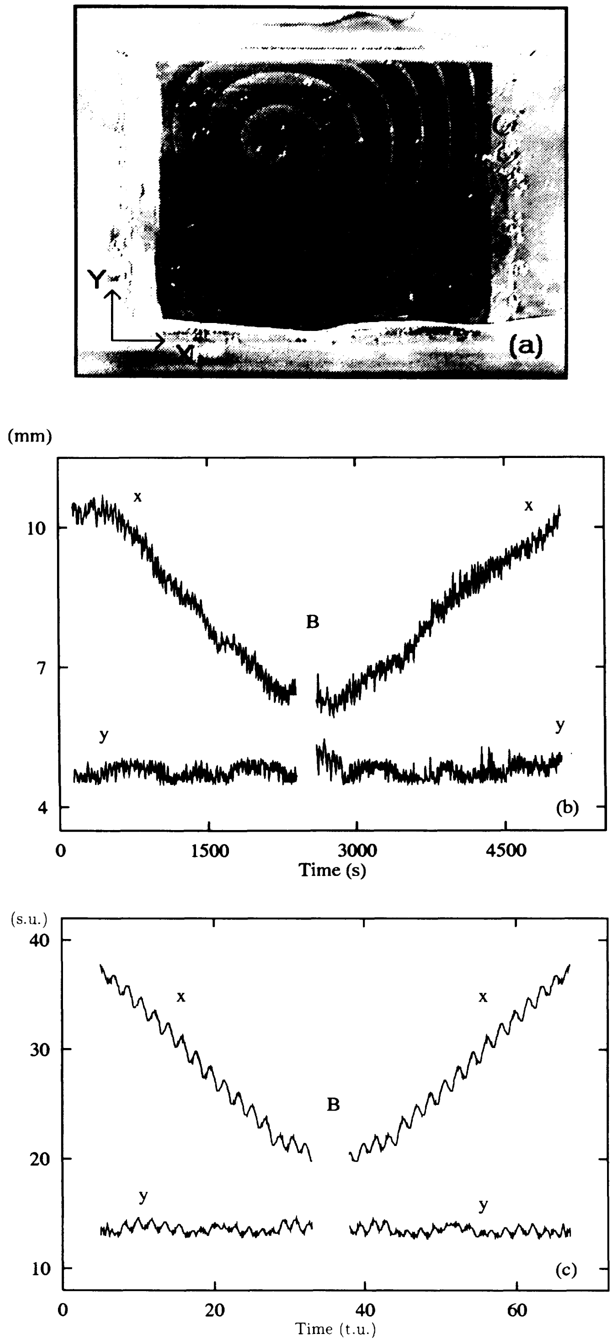

FIG. 1. Parametric resonance of a vortex. (a) The vortex 40 min after the start of the experiment; its displacement is shown by the arrow. (b) Coordinates of the vortex tip in this experiment. At $t=40$ min, the phase of the electric field was increased by 90° (character B on the graph), and the drift direction changed to the opposite one. (c) Numerical simulation on the Oregonator model (1). Dimensionless time units (t.u.) and space units (s.u.) are used throughout. At $t=35$, the phase of the electric field was increased by 90°, resulting in a change of the drift direction ($D_u=1$, $D_v=0.6$, $q=0.002$, $f=1.4$, and $\varepsilon=0.01$, pulse amplitude $ME=2$, and duration 0.12). In both cases, the vortex drifted in the direction parallel to the electric field. Under different conditions drift can occur in a different direction (see text). (For simplicity, D_v was set equal to 0.6 in order to avoid the meandering of the vortex tip.)

$F(u,v)=u-u^2-fv(u-q)/(u+q)$ and $\phi(u,v)=u-v$, and the explicit Euler method was used for integration with $\Delta t=0.001$ and $\Delta x=0.2$ on a squared 200×200 grid.

Figure 2 shows the dependence of the drift velocity on the frequency of the alternating electric field. The largest peak is situated exactly at $f=2f_{@}$. The second-order resonances appear at $f=4f_{@}$ and $0.8f_{@}$. For a nonresonant frequency, a vortex drifts along a circle [Fig. 2(b)], the diameter of a circle increasing inversely proportional to the frequency detuning.

For a parametric resonant frequency, a vortex drifts along a straight line. The direction of the vortex drift is changing continuously between 0 and 360° when the phase shift between electric-field alternations and vortex

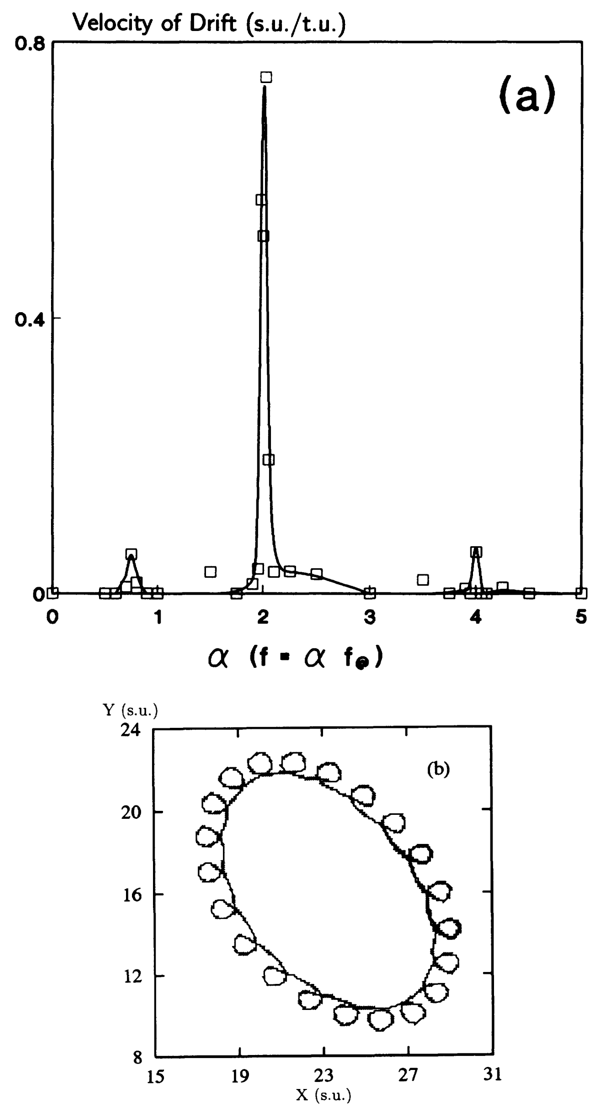

FIG. 2. Frequency dependence of the vortex movement. (a) The resonant curve for the drift velocity. The dimensionless frequency of electric field α ($f=\alpha f_{@}$) is shown in the abscissa. (b) Pattern of a vortex drift at nonresonant frequency $f=2.05f_{@}$ (numerical simulation, same parameters as in Fig. 1).

rotation is changed between 0 and 180° (Fig. 3).

A change in the chirality of a vortex affects its behavior. The simplest way to understand the role of the chirality is to find a coordinate transformation which changes the chirality of a vortex without changing the electric field. For an electric field oriented along the X axis, this transformation is $X \rightarrow X$, $Y \rightarrow -Y$. The phase angle of the rotation is changed by this transformation as $\varphi \rightarrow 180° - \varphi$. It is easily seen now that the behavior shown in Fig. 3(b) can be obtained from that shown in Fig. 3(a) by this transformation.

In conservative systems, parametric resonance is governed by the equation [27]

$$x'' + \omega_0^2 F x = 0 , \quad (2)$$

and forced resonance by the equation

$$x'' + \omega_0^2 x + F = 0 , \quad (3)$$

where F describes parametric pumping of the energy in (2) and external force in (3). Different behavior in both cases is determined by the time symmetry breaking of the equations, and the external influence F is multiplicative in (1) and (2) and additive in (3). This results in the resonance frequency ω_0 in forced oscillations, and $2\omega_0$ in parametrically driven oscillations.

In the experiment with an oscillating electric field, when the tip of the vortex is oriented perpendicular to the electric-field direction, the field influences its normal velocity V_N only. When the tip of the vortex is oriented parallel to the electric field, the field influences only its growing velocity V_g. Thus, due to the rotation of the vortex, the tip is oriented perpendicular (or parallel) to the electric field twice during the period of a vortex rotation, and this is why the resonant frequency is $f = 2f_@$.

The electric field oscillates between positive and negative values, and positive polarity displaces ions in one direction, while negative polarity does so in the opposite direction. Thus the total effect on the ions' movement is zero, and the vortex does not drift. The reason why the vortex drifts under these conditions is that the oscillations of the electric field are synchronized with the rotation of the vortex in such a way that the positive polarity affects only the normal velocity of the vortex V_N, and the negative polarity affects only the growing velocity V_g. As the effect of the electric field on both components of the tip velocity is different, the vortex drifts when $f = 2f_@$ (and this frequency is typical of classical parametric resonance). This resonance can also be interpreted as a parametric one because the alternating electric field affects the parameters of the rotating vortex (in particular, the core size [18,19], which affects the period of oscillations). When $f = f_@$, positive and negative electric pulses are synchronized in such a way that both of them affect only the growing velocity of the tip, so both effects will be compensated for and no drift is observed.

The difference in the resonant behavior of the vortex with periodic illumination [7] and with oscillating electric

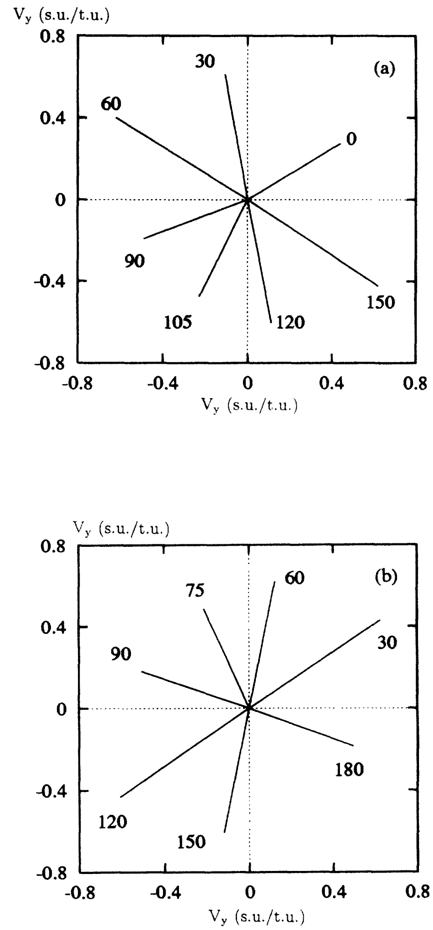

FIG. 3. Hodograph of the parametric resonance drift velocities for vortices of different chirality. (a) Anticlockwise rotating vortex. (b) Clockwise rotating vortex (numerical simulation, same parameters as in Fig. 1). The vector of the drift velocity (the modulus and the angle) for different phase shifts (indicated near each line) is shown. The phase shift was measured as the angle between the northern position of the vortex tip and its position at the moment when the electric field was switched on.

field is due to different symmetries. The homogeneous illumination of the BZ reaction does not break the space symmetry; here a parameter of the medium (excitability) is changed by the light, and the drift of vortices is observed at $f = 1f_@$. The electric field breaks the space symmetry and creates one selected direction. Different resonant frequencies in both cases are due to different symmetries.

It is important to note the difference in parametric resonance behavior in conservative systems and in excitable active media. Energy is not conserved during wave propagation in an excitable medium; a wave propagates due to (chemical) energy stored in the medium. Its amplitude and shape remain constant during propagation, which is not the case for waves in conservative media of dimension greater than one (e.g., solitons may have a constant amplitude only for one-dimensional propagation). For parametric resonance of vortices in an active medium, no pumping of energy is involved. However, the type of symmetry breaking for equations in active media is the same as for parametric resonance in conservative systems.

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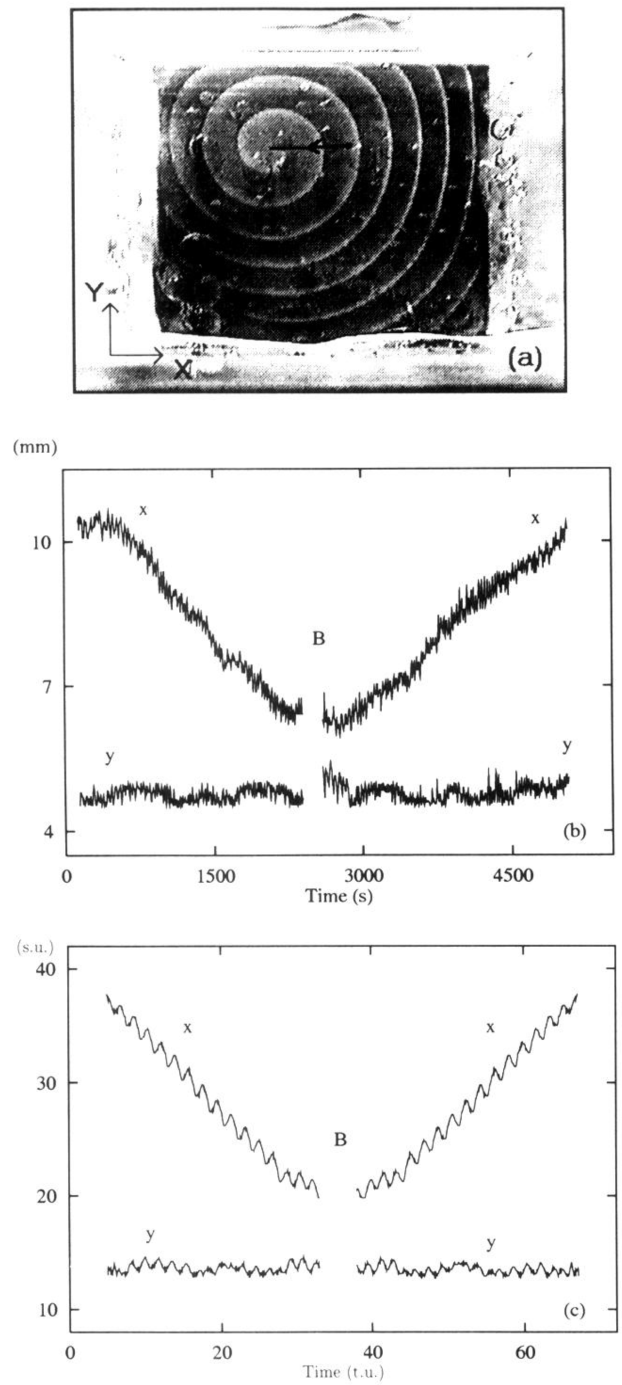

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